QP CODE: 26018595

26018595

Reg No : .....

Name : .....

MSc DEGREE (CSS) SPECIAL REAPPEARANCE EXAMINATION MARCH 2026

Third Semester

M.Sc. MATHEMATICS

CORE COURSE - ME010303 - MULTIVARIATE CALCULUS AND INTEGRAL TRANSFORMS

2024 ADMISSION ONLY

176150AA

Time: 3 Hours Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Find the Fourier Series for $f(x) = 6x, 0 < x < 2\pi$.
2. Define convolution of f and g. Also show by an example that Lebesgue integrability of f and g alone will not give a convolution integral of f and g.
3. Show that if $\mathbf{F}(t) = \mathbf{f}(\mathbf{c} + t\mathbf{u})$, then $\mathbf{F}'(t) = \mathbf{f}'(\mathbf{c} + t\mathbf{u}; \mathbf{u})$, if either derivative exists.
4. Define total derivative of a function $\mathbf{f} : S \to \mathbf{R}^m$, $S \subseteq \mathbf{R}^n$. Show that if $\mathbf{f}$ is differentiable at $\mathbf{c}$, then $\mathbf{f}$ is continuous at $\mathbf{c}$.
5. Give matrix representation for a linear function $T : \mathbf{R}^n \to \mathbf{R}^m$.
6. Define the Jacobian determinant of a function f on R^n
7. Let A be an open set of R^n and assume that $f : A \to R^n$ is continuous and has finite partial derivatives $D_j f_i$ on A. If $f$is one-to-one on A and if $J_f(x) \neq 0$ for each x in A then prove that $f(A)$ is open.
8. Prove that the function $f : R^2 \to R$ defined by $f(x, y) = (y - x^2)(y - 2x^2)$does not have a local maximum or local minimum at $(0, 0)$
9. *Define projection functions in R^n.*
10. Define differential form of order k and explain its elementary properties.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Prove that every real valued and continuous function on a compact interval can be uniformly approximated by a polynomial.
12. State the convolution theorem, using integrals. Also write the relation connecting beta function with gamma function.

13. Find the Jacobian matrix for the function $\mathbf{f}: \mathbf{R}^3 \to \mathbf{R}^3$ defined by the equation $\mathbf{f}(x, y, z) = (sinxcosye^z, sin5xsinysinz, cos5xcosycosz)$.

14. a. Show that if S is an open convex subset of $\mathbf{R}^n$ and $\mathbf{f}: S \to \mathbf{R}^m$, is differentiable at each point of S, then for every vector $\mathbf{a} \in R^m$ and $\mathbf{x}, \mathbf{y} \in S$ there exist $\mathbf{z}$ in the line segment joining $\mathbf{x}$ and $\mathbf{y}$ such that $\mathbf{a}.\{\mathbf{f}(\mathbf{y}) - \mathbf{f}(\mathbf{x})\} = \mathbf{a}.\{f'(\mathbf{z})(\mathbf{y} - \mathbf{x})\}$.
b. If in addition all partial derivatives $D_j f_k$ are bounded on S then show that there exists a constant $A \geq 0$ such that $\|\mathbf{f}(\mathbf{y}) - \mathbf{f}(\mathbf{x})\| \leq A\|\mathbf{y} - \mathbf{x}\|$.

15. For some integer $n \geq 1$, let f have a continuous n^{th} derivative in the open interval (a, b). Also for some interior point cin (a, b), $f'(c) = f''(c) = \ldots\ldots = f^{n-1}(c) = 0$but $f^n(c) \neq 0$. Prove that f has a local maximum at c if n is even and $f^n(c) < 0$ and a local minimum at c if n is even and $f^n(c) > 0$ and if n is odd f cannot have a local maximum or local minimum at c.

16. (a)Define stationary point and saddle point of a function from $R^n \to R$
(b) Find the saddle point of the function $f(x, y) = x^3 - y^3 - 2xy + 6$

17. *For every $f \in C(I^k)$, show that $L(f) = L'(f)$.*

18. *(a)If $\gamma(t) = (acost, bsint)$, $0 \leq t \leq 2\pi$, then find $\int_\gamma xdy$ and $\int_\gamma ydx$.*
(b)If $\Phi(r, \theta, \phi) = (x, y, z)$ where $x = rsin\theta cos\phi$, $y = rsin\theta sin\phi$, $z = rcos\theta$ and D is the $3-$ cell defined by $0 \leq r \leq 1$, $0 \leq \theta \leq \pi$, $0 \leq \phi \leq 2\pi$, then show that $\int_\Phi dx \wedge dy \wedge dz = \frac{4\pi}{3}$.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. State and prove Fourier Integral Theorem.

20. Show that if $\mathbf{g}$ is differentiable at $\mathbf{a}$ and $\mathbf{f}$ is differentiable at $\mathbf{b} = \mathbf{g}(\mathbf{a})$, then the composition function, $\mathbf{h} = \mathbf{f}\, o\, \mathbf{g}$ is differentiable at $\mathbf{a}$ and $\mathbf{h}'(\mathbf{a}) = \mathbf{f}'(\mathbf{b})\, o\, \mathbf{g}'(\mathbf{a})$. Also, express this result in matrix form.

21. State and prove the sufficient condition for differentiability of a function f at the point c

22. *State the partitions of unity theorem. Prove that if $f \in C(R^n)$ and the support of f lies in K, then $f = \sum_{i=1}^{s} \psi_i f$, where each $\psi_i f$ has its support in some V_α.*

(2×5=10 weightage)